

Calculator

An interpreter is a program that understands other programs. Today, we will explore how to build an interpreter for Calculator, a simple language that uses a subset of Scheme syntax.

The Calculator language includes only the four basic arithmetic operations: `+`, `-`, `*`, and `/`. These operations can be nested and can take any numbers of arguments. A few examples of calculator expressions and their corresponding values are shown below.

```
calc> (+ 2 2)
4

calc> (- 5)
-5

calc> (* (+ 1 2) (+ 2 3))
15
```

The reader component of an interpreter parses input strings and represents them as data structures in the implementing language. In this case, we need to represent Calculator expressions as Python objects. To represent numbers, we can just use Python numbers. To represent the names of the arithmetic procedures, we can use Python strings (e.g. `'+'`).

To represent Scheme lists in Python, we will use the `Pair` class. A `Pair` instance holds exactly two elements. Accordingly, the `Pair` constructor takes in two arguments, and to make a list we must nest calls to the constructor and pass in `nil` as the second element of the last pair. Note that in the Python code, `nil` is bound to a special user-defined object that represents an empty list, whereas `nil` in Scheme is actually an empty list.

```
>>> Pair('+', Pair(2, Pair(3, nil)))
Pair('+', Pair(2, Pair(3, nil)))
```

Each `Pair` instance has two instance attributes: `first` and `rest`, which are bound to the first and second elements of the pair respectively.

```

>>> p = Pair('+', Pair(2, Pair(3, nil)))
>>> p.first
'+'
>>> p.rest
Pair(2, Pair(3, nil))
>>> p.rest.first
2

```

Pair is very similar to Link, the class we developed for representing linked lists – they have the same attribute names `first` and `rest` and are represented very similarly. Here's an implementation of what we described:

```

class Pair:
    """Represents the built-in pair data structure in Scheme."""
    def __init__(self, first, rest):
        self.first = first
        if not scheme_valid_cdrp(rest):
            raise SchemeError("cdr can only be a pair, nil, or a
                promise but was {}".format(rest))
        self.rest = rest

    def map(self, fn):
        """Maps fn to every element in a list, returning a new
        Pair.

        >>> Pair(1, Pair(2, Pair(3, nil))).map(lambda x: x * x)
        Pair(1, Pair(4, Pair(9, nil)))
        """
        assert isinstance(self.rest, Pair) or self.rest is nil, \
            "rest element in pair must be another pair or nil"
        return Pair(fn(self.first), self.rest.map(fn))

    def __repr__(self):
        return 'Pair({}, {})'.format(self.first, self.rest)

```

```
class nil:
    """Represents the special empty pair nil in Scheme."""
    def map(self, fn):
        return nil
    def __getitem__(self, i):
        raise IndexError('Index out of range')
    def __repr__(self):
        return 'nil'

nil = nil() # this hides the nil class *forever*
```

Questions

Q1: Using Pair

Answer the following questions about a `Pair` instance representing the Calculator expression `(+ (- 2 4) 6 8)`.

Write out the Python expression that returns a `Pair` representing the given expression:

What is the operator of the call expression?

If the `Pair` you constructed in the previous part was bound to the name `p`, how would you retrieve the operator?

What are the operands of the call expression?

If the `Pair` you constructed was bound to the name `p`, how would you retrieve a list containing all of the operands?

How would you retrieve only the first operand?

Q2: New Procedure

Suppose we want to add the `//` operation to our Calculator interpreter. Recall from Python that `//` is the floor division operation, so we are looking to add a built-in procedure `//` in our interpreter such that `(// dividend divisor)` returns `dividend // divisor`. Similarly we handle multiple inputs as illustrated in the following example `(// dividend divisor1 divisor2 divisor3)` evaluates to `((dividend // divisor1) // divisor2) // divisor3`. For this problem you can assume you are always given at least 1 divisor. Also for this question do you need to call `calc_eval` inside `floor_div`? Why or why not?

```
calc> (// 1 1)
1
calc> (// 5 2)
2
calc> (// 28 (+ 1 1) 1)
14
```

```

def calc_eval(exp):
    if isinstance(exp, Pair): # Call expressions
        return calc_apply(calc_eval(exp.first), exp.rest.map(
            calc_eval))
    elif exp in OPERATORS:   # Names
        return OPERATORS[exp]
    else:                    # Numbers
        return exp
def floor_div(expr):
    """ YOUR CODE HERE """

# Assume OPERATORS['//'] = floor_div is added for you in the code
# You can use more space on the back if you want

```

Q3: New Form

Suppose we want to add handling for comparison operators `>`, `<`, and `=` as well as `and` expressions to our Calculator interpreter. These should work the same way they do in Scheme.

```

calc> (and (= 1 1) 3)
3
calc> (and (+ 1 0) (< 1 0) (/ 1 0))
#f

```

- i. Are we able to handle expressions containing the comparison operators (such as `<`, `>`, or `=`) with the existing implementation of `calc_eval`? Why or why not?
- ii. Are we able to handle `and` expressions with the existing implementation of `calc_eval`? Why or why not?

Hint: Think about the rules of evaluation we've implemented in `calc_eval`. Is anything different about `and`?

- iii. Now, complete the implementation below to handle `and` expressions. You may assume the conditional operators (e.g. `<`, `>`, `=`, etc) have already been implemented for you.

```
def calc_eval(exp):
    if isinstance(exp, Pair):
        if _____: # and expressions
            return eval_and(exp.rest)
        else:
            # Call expressions
            return calc_apply(calc_eval(exp.first), exp.rest.map(
calc_eval))
    elif exp in OPERATORS: # Names
        return OPERATORS[exp]
    else:
        # Numbers
        return exp

def eval_and(operands):
    """ YOUR CODE HERE """

# You can use more space on the back if you want
```

Q4: Saving Values

In the last few questions we went through a lot of effort to add operations so we can do most arithmetic operations easily. However it's a real shame we can't store these values. So for this question let's implement a `define` special form that saves values to variable names. This should work like variable assignment in Scheme; this means that you should expect inputs of the form `(define <variable_name> <value>)` and these inputs should return the symbol corresponding to the variable name.

```
calc> (define a 1)
a
calc> a
1
```

This is a more involved change. Here are the 4 steps involved: 1. Add a `bindings` dictionary that will store the names and corresponding values of variables as key-value pairs of the dictionary. 2. Identify when the `define` form is given to `calc_eval`. 3. Allow variables to be looked up in `calc_eval`. 4. Write the function `eval_define` which should actually handle adding names and values to the `bindings` dictionary.

We've done step 1 for you. Now you'll do the remaining steps in the code below.

```

bindings = {}
def calc_eval(exp):
    if isinstance(exp, Pair):
        if _____: # and expressions[paste your
            answer from the earlier]
            return eval_and(exp.rest)
        elif _____: # define expressions
            return eval_define(exp.rest)

        else:
            # Call expressions
            return calc_apply(calc_eval(exp.first), exp.rest.map(
                calc_eval))
    elif _____: # Looking up variables
        "*** YOUR CODE HERE ***"

    elif exp in OPERATORS:
        # Looking up procedures
        return OPERATORS[exp]
    else:
        # Numbers
        return exp

def eval_define(expr):
    "*** YOUR CODE HERE ***"

# You can use more space on the back if you want

```

Q5: Counting Eval and Apply

How many calls to `calc_eval` and `calc_apply` would it take to evaluate each of the following Calculator expressions?

```
scm> (+ 1 2)
```

For this particular prompt please list out the inputs to `calc_eval` and `calc_apply`.

```
scm> (+ 2 4 6 8)
```

```
scm> (+ 2 (* 4 (- 6 8)))
```

```
scm> (and 1 (+ 1 0) 0)
```


Q6: From Pair to Calculator

Write out the Calculator expression with proper syntax that corresponds to the following Pair constructor calls.

```
>>> Pair('+', Pair(1, Pair(2, Pair(3, Pair(4, nil)))))
```

```
>>> Pair('+', Pair(1, Pair(Pair('*', Pair(2, Pair(3, nil))), nil)))
```